GWs from first-order phase transitions

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arXiv:1705.01783 and references therein
What's next? LISA

- LISA: three arms (six laser links), 2.5 M km separation
- Launch as ESA’s third large-scale mission (L3) in (or before) 2034
- Officially adopted on 20.6.2017
From the LISA proposal:

SI7.2: Measure, or set upper limits on, the spectral shape of the cosmological stochastic GW background

OR7.2: Probe a broken power-law stochastic background from the early Universe as predicted, for example, by first order phase transitions [21] (other spectral shapes are expected, for example, for cosmic strings [22] and inflation [23]). Therefore, we need the ability to measure \( \Omega = 1.3 \times 10^{-11} \left( f / 10^{-4} \text{ Hz} \right)^{-1} \) in the frequency ranges \( 0.1 \text{ mHz} < f < 2 \text{ mHz} \) and \( 2 \text{ mHz} < f < 20 \text{ mHz} \), and \( \Omega = 4.5 \times 10^{-12} \left( f / 10^{-2} \text{ Hz} \right)^{3} \) in the frequency ranges \( 2 \text{ mHz} < f < 20 \text{ mHz} \) and \( 0.02 < f < 0.2 \text{ Hz} \).
First order thermal phase transition:

1. Bubbles nucleate and grow
2. Expand in a plasma - create shock waves
3. Bubbles + shocks collide - violent process
4. Sound waves left behind in plasma
5. Turbulence; expansion
Thermal phase transitions

- **Standard Model is a crossover**
  
  Kajantie et al.; Karsch et al.; ...

- **First order possible in extensions**
  
  (xSM, 2HDM, ...)

  Andersen et al., Kozaczuk et al., Carena et al.,
  Bödeker et al., Damgaard et al., Ramsey-Musolf et al.,
  Cline and Kainulainen, ...

- **Baryogenesis?**

- **GW PS ↔ model information?**
What the metric sees at a thermal phase transition

- Bubbles nucleate and expand, shocks form, then:
  1. $h^2 \Omega_\phi$: Bubbles + shocks collide - 'envelope phase'
  2. $h^2 \Omega_{\text{sw}}$: Sound waves set up - 'acoustic phase'
  3. $h^2 \Omega_{\text{turb}}$: [MHD] turbulence - 'turbulent phase'

- Sources add together to give observed GW power:
  $$h^2 \Omega_{\text{GW}} \approx h^2 \Omega_\phi + h^2 \Omega_{\text{sw}} + h^2 \Omega_{\text{turb}}$$
Equation of motion is (schematically)
Liu, McLerran and Turok; Prokopec and Moore

\[ \partial_\mu \partial^\mu \phi + V'_\text{eff} (\phi, T) + \sum_i \frac{dm_i^2}{d\phi} \int \frac{d^3 k}{(2\pi)^3 2E_i} \delta f_i(k, x) = 0 \]

- \( V'_\text{eff} (\phi) \): gradient of finite-\( T \) effective potential
- \( \delta f_i(k, x) \): deviation from equilibrium phase space density of \( i \)th species
- \( m_i \): effective mass of \( i \)th species:
  - Leptons: \( m^2 = y^2 \phi^2 / 2 \)
  - Gauge bosons: \( m^2 = g_w^2 \phi^2 / 4 \)
  - Also Higgs and pseudo-Goldstone modes
Put another way:

\[
\frac{\partial \mu T^{\mu\nu}}{} - \int \frac{d^3 k}{(2\pi)^3} f(k) F^\nu = 0
\]

This equation is the realisation of this idea:
Yet another interpretation:

\[
\frac{\partial_{\mu} T^{\mu\nu}}{\partial_{\mu} T^{\mu\nu}} - \int \frac{d^3 k}{(2\pi)^3} f(k) F^{\nu} = 0
\]

i.e.:

\[
\partial_{\mu} T^{\mu\nu}_\phi + \partial_{\mu} T^{\mu\nu}_{\text{fluid}} = 0
\]

We will return to this later!
Envelope approximation
Envelope approximation

Kosowsky, Turner and Watkins; Kamionkowski, Kosowsky and Turner

- Thin, hollow bubbles, no fluid
- Stress-energy tensor $\propto R^3$ on wall
- Solid angle: overlapping bubbles $\rightarrow$ GWs
- Simple power spectrum:
  - One length scale (average radius $R_*$)
  - Two power laws ($\omega^3, \sim \omega^{-1}$)
  - Amplitude
    $\Rightarrow$ 4 numbers define spectral form

**NB:** Used to be applied to shock waves (fluid KE), now only use for bubble wall (field gradient energy)
Envelope approximation

4-5 numbers parametrise the transition:

- $\alpha_{T\ast}$, vacuum energy fraction
- $\nu_w$, bubble wall speed
- $\kappa_\phi$, conversion 'efficiency' into gradient energy $(\nabla \phi)^2$
- Transition rate:
  - $H_\ast$, Hubble rate at transition
  - $\beta$, bubble nucleation rate

→ ansatz for $h^2\Omega_\phi$

[only matters for near-vacuum/runaway transitions]
Envelope approximation
Coupled field and fluid system

Ignatius, Kajantie, Kurki-Suonio and Laine

- Scalar $\phi$ and ideal fluid $u^\mu$:
  - Split stress-energy tensor $T^{\mu\nu}$ into field and fluid bits
    \[
    \partial_\mu T^{\mu\nu} = \partial_\mu (T_\phi^{\mu\nu} + T_{\text{fluid}}^{\mu\nu}) = 0
    \]
  - Parameter $\eta$ sets the scale of friction due to plasma
    \[
    \partial_\mu T_\phi^{\mu\nu} = \tilde{\eta} \frac{\phi^2}{T} u^\mu \partial_\mu \phi \partial^\nu \phi
    \]
    \[
    \partial_\mu T_{\text{fluid}}^{\mu\nu} = -\tilde{\eta} \frac{\phi^2}{T} u^\mu \partial_\mu \phi \partial^\nu \phi
    \]
  - $V(\phi, T)$ is a 'toy' potential tuned to give latent heat $\mathcal{L}$
  - $\beta \leftrightarrow$ number of bubbles; $\alpha_{T_*} \leftrightarrow \mathcal{L}$, $\nu_{\text{wall}} \leftrightarrow \tilde{\eta}$

Begin in spherical coordinates:
what sort of solutions does this system have?
Velocity profile development: small $\tilde{\eta} \Rightarrow$ detonation (supersonic wall)
Velocity profile development: large $\tilde{\eta} \Rightarrow$ deflagration (subsonic wall)
\( \nu_w \) as a function of \( \tilde{\eta} \)

Cutting [Masters dissertation]
Simulation slice example
Velocity power spectra and power laws

Fast deflagration

Detonation

- Weak transition: $\alpha_{T^*} = 0.01$
- Power law behaviour above peak is between $k^{-2}$ and $k^{-1}$
- “Ringing” due to simultaneous nucleation, unimportant
GW power spectra and power laws

Fast deflagration

Detonation

- Causal $k^3$ at low $k$, approximate $k^{-3}$ or $k^{-4}$ at high $k$
- Curves scaled by $t$: source until turbulence/expansion

→ power law ansatz for $h^2\Omega_{sw}$
Transverse versus longitudinal modes – turbulence?

- Short simulation; weak transition (small $\alpha$): linear; most power in longitudinal modes $\Rightarrow$ acoustic waves, turbulent
- Turbulence requires longer timescales $R_*/U_f$
- Plenty of theoretical results, use those instead

Kahniashvili et al.; Caprini, Durrer and Servant; Pen and Turok; ...

$\Rightarrow$ power law ansatz for $h^2\Omega_{turb}$
Putting it all together - $h^2 \Omega_{gw}$ arXiv:1512.06239

- Three sources, $\approx h^2 \Omega_\phi$, $h^2 \Omega_{sw}$, $h^2 \Omega_{turb}$
- Know their dependence on $T_*$, $\alpha_T$, $\nu_w$, $\beta$
  
  Espinosa, Konstandin, No, Servant

- Know these for any given model, predict the signal...

(example, $T_* = 100\text{GeV}$, $\alpha_{T_*} = 0.5$, $\nu_w = 0.95$, $\beta/H_* = 10$)
Putting it all together - physical models to GW power spectra

Model \( \rightarrow (T_*, \alpha_{T*}, \nu_w, \beta) \rightarrow \) this plot

... which tells you if it is detectable by LISA (see arXiv:1512.06239)
Detectability from acoustic waves alone

- In many cases, sound waves dominant
- Parametrise by RMS fluid velocity $\bar{U}_f$ and bubble radius $R_*$ (quite easily obtained Espinosa, Konstandin, No and Servant)

Sensitivity plot:
1. Choose your model (e.g. SM, xSM, 2HDM, ...)
2. Dim. red. model Kajantie et al.
3. Phase diagram ($\alpha_{T^*}, T^*$);
   lattice: Kajantie et al.
4. Nucleation rate ($\beta$);
   lattice: Moore and Rummukainen
5. Wall velocities ($v_{\text{wall}}$)
   Moore and Prokopec; Kozaczuk
6. GW power spectrum $\Omega_{gw}$
7. Sphaleron rate

Very leaky, even for SM!
Questions, requests or demands...

- Turbulence
  - MHD or no MHD?
  - Timescales $H_* R_*/U_f \sim 1$, sound waves and turbulence?
  - More simulations needed?

- Interaction with baryogenesis
  - Competing wall velocity dependence of BG and GWs?
  - Sphaleron rates in extended models?

- The best possible determinations for xSM, 2HDM, ΣSM, ...
  - What is the phase diagram?
  - Nonperturbative nucleation rates?