Thermalization in Axion Inflation

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In collaboration with Alessio Notari [arXiv:1706.00373]
Axions in inflation

• Appealing way of realizing large field inflation, their mass is protected by the (discrete) shift symmetry. E.g.: Natural Inflation [Freese, Frieman and Olinto ’90]

\[ \mathcal{L}_\phi = K(\phi) + \Lambda^4(1 + \cos(\phi/f)) \]

• Axions (\(\phi\)) are expected to couple to gauge fields through an axial coupling

\[ \frac{\phi}{f} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad \tilde{F}^{\mu\nu} = \epsilon^{\mu\nu\alpha\beta} \frac{F_{\alpha\beta}}{\sqrt{-g}} \]

where \(\phi\) is the inflaton and \(f\) is the axion decay constant.

• When \(\phi\) develops a VEV, parity is broken and the eom for the massless gauge field (\(A_{\pm}\)) during inflation becomes [Anber and Sorbo 06’]

\[ A''_{\pm}(\eta, k) + \left( k^2 \pm \frac{2k\xi}{\eta} \right) A_{\pm}(\eta, k) = 0, \quad \xi = \frac{\dot{\phi}}{2fH} \]

c.f. talks by P. Adshead, R. Caldwell, A. Maleknejad, M. Peloso, E. Sfakianakis
• **Instability band:** \((8\xi)^{-1} < -k\eta < 2\xi\). If \(\xi \approx \text{constant}\): [Anber and Sorbo 06']

\[
A_k(\eta) \approx \frac{1}{\sqrt{2k}} e^{-ik\eta}, \quad \text{subhorizon}
\]

\[
A_k(\eta) \approx \frac{1}{2\sqrt{\pi k\xi}} e^{\pi \xi}, \quad \text{superhorizon}
\]

• **Phenomenology:**
  - Large loop corrections to \(\zeta\) induced through the coupling \(\xi \zeta F \tilde{F}\)

  2-point function:

  \[
P^\zeta_{1\text{-loop}} = \mathcal{O}(10^{-4}) P_{\text{obs}}^2 e^{4\pi \xi}
\]

  non-Gaussianity:

  \[
f_{NL}^{\text{equi}} |_{1\text{-loop}} = \mathcal{O}(10^{-7}) P_{\text{obs}} e^{6\pi \xi}
\]

  - Large tensor modes, backreaction, preheating, ...

• Observations constraint \(\xi \lesssim 2.5\) \((\xi < 2.2)\) which imposes a lower bound on \(f\).

Instability ⇒ particle production of modes

- Instability band covers subhorizon modes where particle interpretation is meaningful.

- Gauge field effective particle number \( N_\gamma \) per mode \( k \):

\[
\frac{1}{2} + N_\gamma(k) = \frac{\rho_\gamma(k)}{\omega(k)} = \sum_{\text{pol}} \frac{A_k^2 + k^2 A_k^2}{2 \omega(k)} \Rightarrow \begin{cases} 
N_\gamma(k) \approx 0, & k/a \gg H \\
N_\gamma(k) \approx \frac{e^{2\pi \xi}}{8\pi \xi}, & k/a \ll H
\end{cases}
\]

What happens when there are many particles around...?
Scatterings and decays involving $\phi\gamma$ interactions are enhanced by powers of $N_\gamma$

\[
S_{\gamma\gamma \rightarrow \gamma\gamma} = \frac{1}{E_1} \int \prod_{i=2}^{4} \left( \frac{d^3 p_i}{(2\pi)^3(2E_i)} \right) |M_n|^2 (2\pi)^4 \delta^{(4)}(k^\mu + p_2^\mu - p_3^\mu - p_4^\mu) B_{\gamma\gamma \rightarrow \gamma\gamma}(k, p_2, p_3, p_4)
\]

where $B_{\gamma\gamma \rightarrow \gamma\gamma}(p_1, p_2, p_3, p_4)$ contains the phase space factors given by

\[
B_{\gamma\gamma \rightarrow \gamma\gamma}(p_1, p_2, p_3, p_4) = N_\gamma(p_1)N_\gamma(p_2) \left[ 1 + N_\gamma(p_3) \right] \left[ 1 + N_\gamma(p_4) \right] - (p_1 \leftrightarrow p_3, p_2 \leftrightarrow p_4)
\]

and is Bose enhanced by $N_\gamma^3$. 
• Scatterings and decays rates are $\propto N_\gamma^3, N_\gamma^2$. Therefore, when $N_\gamma$ reaches a given threshold then

$$t_{\text{scatterings, decays}} \ll H^{-1} \Rightarrow \text{thermalization}$$

• To estimate the conditions for thermalization we derive, from the eom, Boltzmann-like eqs. for $N_{\gamma+}(k)$, $N_{\gamma-}(k)$ and $N_\phi(k)$:

$$N'_{\gamma+}(k, \eta) = -\frac{4k\xi}{\eta} \frac{\Re[g(k, \eta)]}{|g(k, \eta)|^2 + k^2} (N_{\gamma+}(k, \eta) + 1/2)$$

$$N'_{\gamma-}(k, \eta) \simeq N'_\phi(k, \eta) \simeq 0$$

where $g(k, \eta) = A'(k, \eta)/A(k, \eta)$. 

• Then, add the scatterings and decays.
• Scatterings and decays rates are $\propto N_\gamma^3, N_\gamma^2$. Therefore, when $N_\gamma$ reaches a given threshold then
\[ t_{\text{scatterings, decays}} \ll H^{-1} \implies \text{thermalization} \]

• To estimate the conditions for thermalization we derive, from the eom, Boltzmann-like eqs. for $N_{\gamma+}(k)$, $N_{\gamma-}(k)$ and $N_\phi(k)$:
\[
N'_{\gamma+}(k) = -\frac{4k\xi}{\tau} \frac{\text{Re}[g_A(k, \tau)]}{|g_A(k, \tau)|^2 + k^2} (N_{\gamma+}(k) + 1/2) + S^{++} + S^{+\phi} + D^{+\phi} + S^{+-},
\]
\[
N'_{\gamma-}(k) = -S^{+-},
\]
\[
N'_\phi(k) = -S^{+\phi} - D^{+\phi},
\]

• Solve the system numerically and verify if the distribution approaches a Bose-Einstein distribution.
Numerical results

- Box with $\mathcal{O}(10)$ modes of comoving momentum: $k \in [1, \mathcal{O}(10)]H$.
  Duration of simulation: $\simeq 1$ e-fold, $\{\eta_0 = -2, \eta_f = -1\}$

- Checking thermalization by looking at the average difference to a BE distribution

$$\frac{\Delta N}{N} \equiv \frac{1}{N_{\text{tot}}} \sum_k \frac{N_{\text{norm}}(k) - N_{\text{eq}}(k, T)}{N_{\text{eq}}(k, T)}$$

Left: Change in the particle numbers after thermalization for $f = 0.1H$, $\xi = 2$. Center: Final particle number vs $f/H$ for $\xi = 3.9$ Right: Average difference to Bose-Einstein distribution vs $f/H$ for $\xi = 3.9$.  

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**T**hermalization in Axion Inflation
Numerical results:

- Distribution of particles approaches a BE distribution

- Numerically the system thermalizes when

\[ \xi \gtrsim 0.44 \log \left( \frac{f}{H} \right) + 3.4, \]

- Observations impose \( f\xi/H \gtrsim 10^3 \). This means \( \xi \gtrsim 5.8 \Rightarrow \text{backreacting and non-perturbative regime} \Rightarrow \text{unclear.} \) [RZF et al. 15’, Peloso et al. 16’]

Problem solved if gauge fields belong to the standard model

- More, fixed and unsuppressed interactions (more predictive). More realistic, inflaton has to couple to SM.

- For \( \gamma\psi \) scatterings or gluon self-interactions thermalization requires

\[
\begin{cases}
(\frac{\alpha EM}{2})^2 \left( \frac{H}{k_*} \right)^2 H N_{\gamma\gamma\rightarrow e^-e^+}^2 \gg N_{\gamma\gamma\rightarrow e^-e^+} + H & \Rightarrow \xi \gtrsim 2.9 \\
(\frac{9\pi \alpha_s}{32})^2 \left( \frac{H}{k_*} \right)^2 H N_{gg\rightarrow gg}^3 \gg N_{gg\rightarrow gg} + H & \Rightarrow \xi \gtrsim 2.9
\end{cases}
\]

Under control
• Thermalization shifts particles from horizon size to the UV. At horizon crossing the gauge field particle number is much smaller than in the non-thermal case.

• Effects on $\zeta$ are drastically modified!

• Loop corrections to $\zeta$ correlators are much smaller
  
  • $\langle \zeta^2 \rangle \propto \left( \frac{T}{H} \right)^4 \propto e^{2\pi \xi}$ instead of $e^{4\pi \xi}$ as in the non-thermal case;
  
  • $\langle \zeta^3 \rangle \propto \langle F\tilde{F} \rangle$ is suppressed because parity symmetry tends to be restored

  $$F\tilde{F} \propto (n_{\gamma_+} - n_{\gamma_-}) \rightarrow \xi H^2 T^2$$

• Constraints on $\xi$ become generically weaker.
Is the thermal regime stable?

• Moreover, after thermalization gauge field develops thermal mass $m_T \approx \bar{g} T$

$$A''_\pm + \omega^2_T(k) A_\pm = 0, \quad \omega^2_T(k) = \left( k^2 \pm \frac{2k\xi}{\tau} + \frac{m^2_T}{H^2\tau^2} \right).$$

• If $m_T > \xi H$ the instability disappears and thermal bath redshifts. However, the system should reach an equilibrium (or oscillate around it): if temperature is too small the thermal mass disappears and the instability opens again

- The system should reach an equilibrium temperature which balances the two terms:

$$\omega^2_T(k) \gtrsim 0 \quad \Rightarrow \quad T_{eq} \simeq \frac{\xi H}{\bar{g}}.$$
At $T_{eq}$, $\phi$ is thermalized if $\Gamma(T_{eq}) \gg H$, i.e.:

$$\xi \gg \bar{g} \left( \frac{f}{H} \right)^{4/5}$$

Predictions for thermalized inflaton:

- $P_{\zeta}^{\text{thermal}} \approx \frac{2T}{H} P_{\zeta}^{\text{vac}}$

- $n_s - 1 = -6\epsilon_H + 2\eta + \frac{\dot{\xi}}{H\xi} = -4\epsilon_H + \eta$

- $r = 16\epsilon \frac{H}{2T} = 8\epsilon \frac{\bar{g}}{\xi}$

- $P_{\zeta}^{\text{thermal}}(T_{eq}) = 2.2 \times 10^{-9}$ gives a condition for $\xi(f)$.

- Non-Gaussianity estimated to be

$$f_{NL}^{\text{thermal}} \lesssim d\xi^4 P_{\zeta}^{\text{vac}} \mathcal{O} \left( \frac{T^5}{H^5} \right) \Rightarrow \xi < \mathcal{O}(20 - 100),$$
Conclusions & Future Work

Done:

- Controlled setup where a thermal bath can be sustained during inflation by the instability created due to the axial coupling.
- Couplings are shift symmetric so no thermal mass is generated for $\phi$.
- Predictions are changed: constraints on $\xi$ relaxed, spectral tilt fixed, $r$ reduced (large field inflation (re)compatible with data!)

To be done:

- Confirm that the system reaches, or oscillates around, the equilibrium temperature.
- Improve non-Gaussianity calculations to derive more precise constraints on $\xi$.
- Is the backreacting regime possible?
- Many different features still to study.

Can inflation be ThAI?