Exploring the Ultra Large Scale Structure of the Universe

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Ultra Large Scale Structure
Ultra Large Scale Structure (ULSS)

Theory
large scales = early times

Inflationary statistics:
• Non-gaussianity.
• Tilt and running.
• Features.

Fundamental questions:
• Multiverse?
• GR on large scales?
• Topology, isotropy, voids, and curvature?
Ultra Large Scale Structure (ULSS)

Our best probe: CMB
Ultra Large Scale Structure (ULSS)

Our best probe: CMB

??Hints??

Figure 3. Angular two-point correlation function as observed by Planck [7]. The full black line and the shaded regions are the expectation from 1000 SMICA simulations based on the $\Lambda$CDM model and the 68% and 95% confidence regions. The plot also shows four colored lines that fall on top of each other and represent the result of the Planck analysis of the Commander, SEVEM, NILC and SMICA maps at resolution $N_{\text{side}} = 64$. While the measured two-point correlation is never outside the 95% confidence region, the surprising feature is that we observe essentially no correlations at $\theta < 70^\circ$ and a significant lack of correlations at $\theta > 60^\circ$. An important question is the size of the mask used in the analysis. It has been shown in [37] that most of the large-angle correlations in reconstructed sky maps are between pairs of points at least one of which is in the part of the sky that is most contaminated by the Galaxy. This is in line with the findings of [32], where it was shown that more conservative masking makes the lack of correlation even more significant. This by itself already signifies a violation of isotropy.

Undoubtedly, $S_{1/2}$ is an ad hoc and a posteriori statistic, but it captures naturally the observed feature originally noted in COBE. Several a posteriori "improvements" have been suggested [39, 7]. For example, in order to avoid the argument that $\mu = 1/2$ has been fixed after the fact one might let $\mu$ vary. But now the look elsewhere effect must be taken into account. The Planck team implemented such an analysis which (in our convention) returns global p-values of the order of 2%. However, this global $S_\mu$ statistic addresses a different question, namely how likely is it that there is a lack of correlation for an arbitrary $\mu$. Thus we cannot argue that this statistic is better than $S_{1/2}$, all we can say is that it is different.
Ultra Large Scale Structure (ULSS)

Our best probe: CMB

??Hints??
Ultra Large Scale Structure (ULSS)

cosmic horizon

Our best probe: CMB

??Hints??
cold spot
Ultra Large Scale Structure (ULSS)

Our best probe: CMB

??Hints??
Ultra Large Scale Structure (ULSS)

Our best probe: CMB

??Hints??

These temperature fluctuations are believed to have been generated from quantum fluctuations in the very early Universe [18] by a (nearly) scale-invariant mechanism. The most prominent context is cosmological inflation [19, 20]. If inflation lasts long enough, the spatial geometry of the Universe is generally predicted to be indistinguishable from Euclidean, and the topology of the observable Universe is expected to be trivial (simply connected). Even more importantly, inflation predicts that the CMB temperature fluctuations should be: (i) statistically isotropic, (ii) Gaussian, and (iii) almost scale invariant. It also predicts: (iv) phase coherence of the fluctuations; (v) for the simplest models, a dominance of the so-called adiabatic mode (strictly speaking it is not only adiabatic but also isentropic); and (vi) the non-existence of rotational modes at large scales. Finally, depending on the energy scale of cosmological inflation, there might be (vii) a detectable stochastic background of gravitational waves [21].
Ultra Large Scale Structure (ULSS)

Cosmic Variance:
Projection of 3D field.
Few modes on large scales/finite volume.

Sadly, the primary CMB will not teach us much more**.

** Better low L polarization will help a little.
Ultra Large Scale Structure (ULSS)

- $z=1$
- $z=6$
- $z=1100$
- Cosmic horizon
Ultra Large Scale Structure (ULSS)

The small-scale CMB may tell us a lot about large scales!

Plenty of (potentially observable) things to understand about ULSS on theory front.
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<td>• Kinetic Sunyaev Zel’dovich (kSZ) tomography.</td>
<td>• What are the observable imprints of a highly inhomogeneous pre-inflationary Universe?</td>
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<tr>
<td>• Polarized Sunyaev Zel’dovich (pSZ) tomography.</td>
<td>• Multiverse?</td>
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Possible with impending precision measurements of CMB secondaries and LSS. New predictions for ULSS.
## Outline

### Data
- Kinetic Sunyaev Zel’dovich (kSZ) tomography.
- Polarized Sunyaev Zel’dovich (pSZ) tomography.

Possible with impending precision measurements of CMB secondaries and LSS.

### Theory
- What are the observable imprints of a highly inhomogeneous pre-inflationary Universe?
- Multiverse?

New predictions for ULSS.
The kinetic Sunyaev Zel’dovich (kSZ) effect

kSZ - Scattering of CMB photons by free electrons (after reionization) moving with respect to the CMB rest frame.

Provides a census of the locally observed CMB dipole at the location of each electron.

Provides information off of our past light cone - new information beyond CMB.
The kinetic Sunyaev Zel’dovich (kSZ) effect

The induced temperature anisotropies are given by a line of sight integral:

$$\left. \frac{\Delta T}{T} \right|_{\text{kSZ}} (\hat{n}_e) = -\sigma_T \int_0^{\chi_{re}} d\chi_e \ a_e(\chi_e) \ \bar{n}_e(\chi_e) \ (1 + \delta(\hat{n}_e, \chi_e)) \ \sum_{m=-1}^{1} v_{\text{eff}}^m(\hat{n}_e, \chi_e) Y_{1m}(\hat{n}_e).$$

The kSZ effect depends on the components of the local CMB dipole:

$$v_{\text{eff}}^m(\hat{n}_e, \chi_e) = \int d^2\hat{n} \ \Theta(\hat{n}_e, \chi_e, \hat{n}) Y_{1m}^*(\hat{n})$$

Note that $$v_{\text{eff}}^m(\hat{n}_e, \chi_e \to 0) = a_{1m}^T$$

Many subtleties with dipole.
The polarized Sunyaev Zel’dovich (pSZ) effect

pSZ - Polarization induced by the scattering of CMB photons by free electrons (after reionization).

Provides a census of the locally observed CMB quadrupole at the location of each electron.

Provides information off of our past light cone - new information beyond CMB, different information from kSZ.
The polarized Sunyaev Zel’dovich (pSZ) effect

The induced polarization anisotropies are given by a line of sight integral:

\[(Q \pm iU)^{\text{pSZ}}(\hat{n}_e) = -\frac{\sqrt{6} \sigma_T}{10} \int d\chi_e \, a_e \bar{n}_e(\chi_e)(1 + \delta_e(\hat{n}_e, \chi_e)) \sum_{m=-2}^{2} q^m_{\text{eff}}(\hat{n}_e, \chi_e) \pm 2 Y_{2m}(\hat{n}_e)\]

The pSZ effect depends on the components of the local CMB quadrupole:

\[q^m_{\text{eff}}(\hat{n}_e, \chi_e) = \int d^2 \hat{n} \, \left[ \Theta(\chi_e, \hat{n}_e, \hat{n}) + \Theta^T(\chi_e, \hat{n}_e, \hat{n}) \right] Y^*_{2m}(\hat{n})\]

**Sensitive to both scalars and tensors**

Note that \(q^m_{\text{eff}}(\hat{n}_e, \chi_e \to 0) = a^T_{2m}\)
kSZ and pSZ Tomography

Dipole and quadrupole fields

CMB
kSZ and pSZ Tomography
kSZ and pSZ Tomography

Redshift bins
Define moments of bin-averaged dipole field

$$\bar{v}_\text{eff}^\alpha(\hat{n}) = \sum \bar{v}_\text{eff,}^\alpha, \ell m Y_{\ell m}(\hat{n})$$

Cross correlate with binned density field:

$$\langle \delta(\bar{\chi}_e) \frac{\Delta T}{T} \rangle \bigg|_{\text{kSZ}} \sim \langle \delta \delta v \rangle \sim \bar{v}(\hat{n}_e, \bar{\chi}_e) \langle \delta \delta \rangle(\bar{\chi}_e) + \text{isotropic}$$

Small-scale contributions to the bin-averaged CMB dipole cancel.

Excellent probe of very large scales!

[Zhang and Stebbins, Zhang ’10, Terrana, Harris, MCJ ’16]
kSZ Tomography

Define moments of bin-averaged dipole field

\[ \bar{v}_\alpha^\alpha(\hat{n}) = \sum \bar{v}_\text{eff,}^\alpha,\ell m Y_{\ell m}(\hat{n}) \]

Cross correlate with binned density field:

\[ \langle \delta(\bar{\chi}_e) \frac{\Delta T}{T} \rangle_{kSZ} \sim \langle \delta \delta v \rangle \sim \bar{v}(\hat{n}_e, \bar{\chi}_e) \langle \delta \delta \rangle(\bar{\chi}_e) + \text{isotropic} \]

Small-scale contributions to the bin-averaged CMB dipole cancel.

Primordial dipole = dipole sourced by linear scales.
kSZ Tomography

Tomography - boost the signal, isolate long-wavelength component, break up line of sight integral, pull out 3D information.

The signal:

Power asymmetry

long wavelength modulation of small scale power
Can we detect it?

- Reconstruction of the bin-averaged dipole field via quadratic estimator.

Per-mode signal to noise assuming no foregrounds, full sky, and cosmic variance limited measurements of CMB temperature and the angular matter power spectrum to \( l = 3000 \) in equally spaced redshift bins to redshift 6.

[Terrana, Harris, MCJ ’16, Deutsch, Dimastrogiovanni, MCJ, Münchmeyer, Terrana in prep.]
Can we detect it?

Our local primordial dipole. Bin size sets range of scales included in definition of “primordial”.
Define moments of bin-averaged quadrupole field:

\[ q_{\text{eff}}^{\pm \alpha} (\hat{n}) = \sum a_{\ell m}^{q, \pm \alpha} + 2Y_{\ell m}(\hat{n}) \]

Cross correlate with binned density field:

\[ \langle \delta (\bar{\chi}_e)(Q \pm iU) \rangle \sim \langle \delta \delta q^{\pm} \rangle \sim q^{\pm} (\hat{n}_e, \bar{\chi}_e) \langle \delta \delta \rangle (\bar{\chi}_e) \]

Modulated small-scale E-modes and B-modes.

[As in Dvorkin, Hu, Smith ’09]

E and B mode components of quadrupole field:

\[ a_{\ell m}^{q,E\alpha} = -\frac{1}{2} (a_{\ell m}^{q,+\alpha} + a_{\ell m}^{q,-\alpha}) \]

\[ a_{\ell m}^{q,B\alpha} = -\frac{1}{2i} (a_{\ell m}^{q,+\alpha} - a_{\ell m}^{q,-\alpha}) \]
Can we detect it?

- Reconstruction of the E-mode scalar quadrupole field via quadratic estimator using both E-mode and B-mode polarization.

Per-mode signal to noise assuming no foregrounds, full sky, and cosmic variance limited measurements of CMB E, B and the angular matter power spectrum to \( l = 3000 \) in equally spaced redshift bins to redshift 6.

[Deutsch, MCJ, Münchmeyer, Terrana’17, Deutsch, Dimastrogiovanni, MCJ, Münchmeyer, Terrana in prep.]
Can we detect it?

![Diagram showing local CMB quadrupole](image)

Our local CMB quadrupole. What would we learn if not?
Can we detect it?

- Reconstruction of the E-mode and B-mode tensor quadrupole field via quadratic estimator using both E-mode and B-mode polarization.

Forecast for $r = 0.1$

Alizadeh and Hirata forecast: $r$ of $0.1$ may be achievable.

[Alizadeh, Hirata ’12, Deutsch, Dimastrogiovanni, MCJ, Münchmeyer, Terrana in prep.]
What’s next?

- Remote dipole and remote quadrupole measurements are becoming realistic.

- Theorists: start thinking about what types of early-Universe or large-scale physics this could be useful for.

- Observers: there are certainly ways to optimize future CMB experiments to go after these signals. There may be signals to detect even in S3 CMB experiments and existing LSS surveys.
Outline

**Data**

- Kinetic Sunyaev Zel’’dovich (kSZ) tomography.
- Polarized Sunyaev Zel’’dovich (pSZ) tomography.

Possible with impending precision measurements of CMB secondaries and LSS.

**Theory**

- What are the observable imprints of a highly inhomogeneous pre-inflationary Universe?
- Multiverse?

New predictions for ULSS.
Multiverse?

- The eternally inflating Multiverse can in some cases have directly observable consequences.

  [Aguirre, MCJ, Shomer ’07]

  Our bubble does not evolve in isolation....

The collision of our bubble with others provides an observational test of eternal inflation.
How do we find the signature of bubble collisions?

**Figure 1.** A schematic of the path from a scalar field Lagrangian to the comoving curvature perturbation. One begins by specifying a scalar-field potential with two or more vacua. With only two vacua (upper left panel), only collisions between identical bubbles are possible. Region 1 on the potential describes the properties of the bubble wall, while regions 2 and 3 describe the cosmological evolution inside of each bubble as depicted in the upper right panel. Simulations are performed to construct the full collision spacetime in a set of global coordinates. Using data from the simulation, we reconstruct the perturbed FRW metric inside the observation bubble by evolving a set of geodesics through the simulation (lower right panel). A gauge transformation then allows us to extract the comoving curvature perturbation $R$ late in the inflationary epoch. Evolving the comoving curvature perturbation, the observable universe is split into a region that is, and a region that is not, affected by the collision. The lower left panel depicts the surface of last scattering inside the observation bubble in a reference frame where the observer is at the origin of coordinates. The collision boundary follows a line of constant $\xi$ (this coordinate is defined by the metric Eq. 2.2).

This observer's past light cone intersects the collision boundary, and would map on to a disc on their CMB sky.

The same initial surface, we simply sum their field values, assuming that the overlap is small enough such that they do not affect each other's shape. We then compute metric variables from the constraints. This procedure is valid because the geometry approaches an anisotropic foliation of flat space at early enough times. We designate one bubble as the observation bubble — it must contain a phenomenologically viable slow-roll inflationary cosmology — and the other bubble as the collision bubble. We then simulate the collision using the fully general-relativistic equations of motion on an adaptive grid, which is necessary to accurately resolve the bubble walls and the collision shock fronts. The simulation coordinates are defined in terms of the false vacuum Hubble parameter $H_F$, with an $SO(2,1)$ symmetric metric

$$H_F^2 ds^2 = \xi(N, x)^2 dN^2 + a(N, x)^2 \cosh^2 N d^2 x + \sinh^2 N (d^2 + \sinh^2 N d^2') \,.$$ (2.1)

[Wainwright, MCJ, Peiris, Aguirre, Lehner, Liebling ’12,’14,’16]
What would it look like?

[Feeney, Elsner, MCJ, Peiris ’16]
CMB constraints

CMB constraints almost CV limited.
Can we do better?

- Constraints from kSZ tomography: [Zhang, MCJ ’15]
Grischuk-Zel’dovich effect in full GR

[Braden, MCJ, Peiris, Aguirre, arxiv:1604.04001]

Posterior PDF on pre-inflationary inhomogeneities given the observed CMB quadrupole from a large ensemble of simulations of inhomogeneous inflationary Universes in full GR.

FIG. 5: The posterior probability distribution for our model parameters $\log A_0$, $\log H_1 L_{\text{obs}}$, and $\log \hat{C}_2^{\text{obs}}$. The dashed curves are the same contours if we model $\log A_0 \geq 1$. The blue dotted line indicates the initial amplitude at which $\hat{C}_2^{\text{obs}}$ begins to become non-Gaussian. The important feature for the analysis we presented is the emergence of a strong peak in the distribution at the true physical distance to the last scattering surface, rather than the physical distance measured in the locally defined FRW background. Furthermore, they are generated using Eq. (17) for both the numerically generated ($\sigma = 0.1$) and Gaussian ($\sigma = 0.6$) units.
Ultra Large Scale Structure (ULSS)

The small-scale CMB may tell us a lot about large scales!

Plenty of (potentially observable) things to understand about ULSS on theory front.
Thanks!
We've demonstrated that we can approximate the correlation in eq. (39) as 
ensemble average in eq. (39) should only be taken over small scales, leaving large scales as a fixed deterministic field.

Captures our signal, the sub-dominant small scale isotropic component contributes to the noise, computed in the
smaller than the dominant anisotropic term

Obtained a statistically isotropic contribution our desired signal.

In (42), which gives rise to a long wavelength modulation of small-scale power.

Small in comparison to the primary CMB. In our analysis, we will exclude these terms by imaging that we have filtered

The quadrupole is a very long wavelength field, and it is thus safe to neglect it's short wavelength component. We

The decomposition in (40) implies a similar long-short split for the quadrupole and density fields, valid in the linear

FIG. 5:

Redshift bin info

Redshift is displayed as a function of
The kinetic Sunyaev Zel’dovich (kSZ) effect

The induced temperature anisotropies are given by a line of sight integral:

\[
\frac{\Delta T}{T}_{\text{kSZ}} (\hat{n}_e) = -\sigma_T \int_0^{\chi_{\text{re}}} d\chi_e \, a_e(\chi_e) \, \bar{n}_e(\chi_e) \, (1 + \delta(\hat{n}_e, \chi_e)) \, v_{\text{eff}}(\hat{n}_e, \chi_e)
\]

The effective velocity is the locally observed CMB dipole projected along the line of sight:

\[
v_{\text{eff}}(\hat{n}_e, \chi_e) = \frac{3}{4\pi} \int d^2\hat{n} \, \Theta(\hat{n}_e, \chi_e, \hat{n}) \, (\hat{n} \cdot \hat{n}_e)
\]

- **Sachs-Wolfe**
  \[
  \left(2D_\Psi(\chi_{\text{dec}}) - \frac{3}{2}\right) \Psi_i(r_{\text{dec}})
  \]

- **Integrated Sachs-Wolfe**
  \[
  2 \int_{a_{\text{dec}}}^{a_e} \frac{d\Psi}{da}(r(a), a) da
  \]

- **Doppler**
  \[
  \hat{n} \cdot [v(r_e, \chi_e) - v(r_{\text{dec}}, \chi_{\text{dec}})]
  \]
Fourier kernel for effective velocity

\[ v_{\text{eff}}(\hat{n}_e, \chi_e) = i \int \frac{d^3 k}{(2\pi)^3} T(k) \tilde{\Psi}_i(k) \left[ \mathcal{K}_d(k, \chi_e) + \mathcal{K}_{\text{SW}}(k, \chi_e) + \mathcal{K}_{\text{ISW}}(k, \chi_e) \right] P_1(\hat{k} \cdot \hat{n}_e) e^{i \chi_e k \cdot \hat{n}_e} \]

\[ \mathcal{K}_d(k, \chi_e) \equiv k D_v(\chi_{\text{dec}}) j_0(k \Delta \chi_{\text{dec}}) - 2 k D_v(\chi_{\text{dec}}) j_2(k \Delta \chi_{\text{dec}}) - k D_v(\chi_e) \]

\[ \mathcal{K}_{\text{SW}}(k, \chi_e) \equiv 3 \left( 2 D_\Psi(\chi_{\text{dec}}) - \frac{3}{2} \right) j_1(k \Delta \chi_{\text{dec}}) \]

\[ \mathcal{K}_{\text{ISW}}(k, \chi_e) \equiv 6 \int_{a_{\text{dec}}}^{a_e} da \frac{dD_\Psi}{da} j_1(k \Delta \chi(a)) \]
The kinetic Sunyaev Zel’’dovich (kSZ) effect

kSZ is the dominant frequency independent contribution to the CMB on small angular scales.

First detections made through pairwise cluster statistic (ACT 2012) and binned power spectrum (SPT 2014).

\[ \hat{n} \cdot [v(r_e, \chi_e) - v(r_{dec}, \chi_{dec})] \]
The polarized Sunyaev Zel’dovich (pSZ) effect

The induced polarization anisotropies are given by a line of sight integral:

\[
(Q \pm iU)^{pSZ}(\hat{n}_e) = -\frac{\sqrt{6} \sigma_T}{10} \int d\chi_e \ a_e \tilde{n}_e(\chi_e)(1 + \delta_e(\hat{n}_e, \chi_e)) \sum_{m=-2}^{2} q_{\text{eff}}^m(\hat{n}_e, \chi_e) \pm 2Y_{2m}(\hat{n}_e)
\]

The effective quadrupole is the locally observed CMB quadrupole for each electron:

\[
q_{\text{eff}}^m(\hat{n}_e, \chi_e) = \int_{\Omega} d^2\hat{n} \ \Theta(\chi_e, \hat{n}_e, \hat{n}) \ Y_{2m}^*(\hat{n})
\]

\[
\begin{align*}
\text{Sachs-Wolfe} & \quad \left(2D_{\Psi}(\chi_{\text{dec}}) - \frac{3}{2}\right) \Psi_i(\mathbf{r}_{\text{dec}}) \\
\text{Integrated Sachs-Wolfe} & \quad 2 \int_{a_{\text{dec}}}^{a_e} \frac{d\Psi}{da}(\mathbf{r}(a), a) da \\
\text{Doppler} & \quad -\hat{n} \cdot \mathbf{v}(\mathbf{r}_{\text{dec}}, \chi_{\text{dec}})
\end{align*}
\]
Fourier kernel for quadrupole field

\[ q_{\text{eff}}^{m}(\hat{n}_e, \chi_e) = \int \frac{d^3k}{(2\pi)^3} \tilde{Y}_i(k) T(k) \left[ G_{SW} + G_{ISW} + G_{Doppler} \right] Y_{2m}^*(\hat{k}) e^{i\chi_e \hat{k} \cdot \hat{n}_e} \]

\[ G_{SW}(k, \chi_e) = -4\pi \left( 2D_{\Psi}(\chi_{\text{dec}}) - \frac{3}{2} \right) j_2(k\Delta \chi_{\text{dec}}), \]

\[ G_{ISW}(k, \chi_e) = -8\pi \int_{a_{\text{dec}}}^{a_e} da \frac{dD_{\Psi}}{da} j_2(k\Delta \chi(a)), \]

\[ G_{Doppler}(k, \chi_e) = \frac{4\pi}{5} kD_v(\chi_{\text{dec}}) \left[ 3j_3(k\Delta \chi_{\text{dec}}) - 2j_1(k\Delta \chi_{\text{dec}}) \right] \]

![Graph showing the Fourier kernel for quadrupole field](image)
The polarized Sunyaev Zel’dovich (pSZ) effect

The pSZ effect on small angular scales is subdominant to lensing.

But pSZ is highly non-gaussian, with hot spots in non-linear structure (e.g. clusters).
kSZ and pSZ tomography

kSZ and pSZ are both small signals, and both involve a line of sight integral. kSZ involves both large-scale and small-scale information.

Tomography - boost the signal, isolate long-wavelength component, pull out 3D information.

\[
\left\langle \frac{\Delta T}{T} \right\rangle_{\text{kSZ}} (\mathbf{n}_e) \delta(\mathbf{n}^\prime_e, \bar{\chi}_e) = \sigma_T \int d\chi_e \ a(\chi_e) \ \bar{n}_e(\chi_e) \ v_{\text{eff}}(\mathbf{n}_e, \chi_e) \int d\chi'_e \ W(\chi'_e, \bar{\chi}_e) \\
\times \left\langle \delta(\mathbf{n}_e, \chi_e) \right\rangle \delta(\mathbf{n}^\prime_e, \chi'_e) + \text{isotropic.}
\]

\[
\left\langle (Q + iU)^{\text{pSZ}}(\mathbf{n}_e) \delta(\mathbf{n}^\prime_e, \bar{\chi}_e) \right\rangle = -\frac{\sqrt{6} \ \sigma_T}{10} \int d\chi_e \ a_e \ \bar{n}_e(\chi_e) \sum_{m=-2}^{2} \ q^m_{\text{eff}}(\mathbf{n}_e, \chi_e) \ \pm 2 \ Y_{2m} \ (\mathbf{n}_e) \\
\times \int d\chi'_e \ W(\chi'_e, \bar{\chi}_e) \left\langle \delta(\mathbf{n}_e, \chi_e) \right\rangle \delta(\mathbf{n}^\prime_e, \chi'_e) + \text{isotropic}
\]

Binned electron density field: \( \delta(\mathbf{n}_e, \bar{\chi}_e) = \int d\chi_e W(\chi_e, \bar{\chi}_e) \delta(\mathbf{n}_e, \chi_e) \)
kSZ and pSZ tomography

The anisotropic component of the cross-correlation can be characterized by power multipoles:

$$b_{LM}(\bar{\chi}_e) = \int d^2 \hat{n}_e \ Y_{LM}^*(\hat{n}_e) \left\langle \frac{\Delta T}{T} \right|_{\text{kSZ}} (\hat{n}_e) \ \delta(\hat{n}_e, \bar{\chi}_e) \right\rangle$$

$$e_{LM}(\bar{\chi}_e) = \int d^2 \hat{n}_e \ \pm 2 Y_{LM}^*(\hat{n}_e) \left\langle (Q \pm iU)^{pSZ}(\hat{n}_e) \delta(\hat{n}_e, \bar{\chi}_e) \right\rangle$$
kSZ and pSZ tomography

The power multipoles are directly related to the monopole and quadrupole fields:

\[ b_{LM} = \sigma_T \int d\chi_e \ a(\chi_e) \ \bar{n}_e(\chi_e) \ a^v_{LM}(\chi_e) \int d\chi'_e \ W(\chi'_e, \bar{\chi}_e) \langle \delta(\hat{n}_e, \chi_e) \ \delta(\hat{n}'_e, \chi'_e) \rangle \]

\[ v_{\text{eff}}(\hat{n}_e, \chi_e) = \sum_{\ell,m} a^v_{\ell m}(\chi_e) Y_{\ell m}(\hat{n}_e) \]

\[ e_{LM}(\bar{\chi}_e) = -\frac{\sqrt{6} \sigma_T}{10} \int d\chi_e \ a_e \ \bar{n}_e \ a^q_{LM}(\chi_e) \int d\chi'_e \ W(\chi'_e, \bar{\chi}_e) \left\langle \delta_e(\hat{n}_e, \chi_e) \ \delta^*(\hat{n}'_e, \chi'_e) \right\rangle \]

\[ \tilde{q}^\pm_{\text{eff}}(\hat{n}_e, \chi_e) = \sum_{\ell m} a^q_{\ell m}(\chi_e) \pm 2 Y_{\ell m}(\hat{n}_e) \]

\[ \hat{q}^\pm_{\text{eff}}(\hat{n}_e, \chi_e) \equiv \sum_{m=-2}^{2} q^m_{\text{eff}}(\hat{n}_e, \chi_e) \pm 2 Y_{2m}(\hat{n}_e) \]
Can we detect it in the near future?

Crazy to expect this?

$$\theta_{\text{FWHM}} = (\frac{8 \ln 2}{\ell_{\text{max}}(\ell_{\text{max}} + 1)})^{1/2}$$

$$N_{\ell}^{\text{CMB}} = \left(\sigma_N \theta_{\text{FWHM}}\right)^2$$

$$N_{g}^{\text{gg}}(\bar{\chi}_e) = \frac{1}{N_g(\bar{\chi}_e)}$$

<table>
<thead>
<tr>
<th>$\ell_{\text{max}}$</th>
<th>$\theta_{\text{FWHM}}$ (arcmin)</th>
<th>$\sigma_N$, $\sigma_F$ (μK $\theta_{\text{FWHM}}^{-1}$)</th>
<th>$N_g(\bar{\chi}_e)$ (arcmin$^{-2}$)</th>
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</thead>
<tbody>
<tr>
<td>3000</td>
<td>2.7</td>
<td>3.7</td>
<td>14, 26, 82, 255, 697, 1661</td>
</tr>
<tr>
<td>5000</td>
<td>1.6</td>
<td>3.0</td>
<td>45, 62, 155, 500, 1700, 5000</td>
</tr>
</tbody>
</table>

1 – 3 arcmin (CMB stage 4)

1 μK arcmin$^{-1}$ (CMB stage 4)

$N_g = 130$ (LSST)

$N_g = 30$ (Euclid)

$N_g = 500$ (Hubble DF)

$N_g = 1700$ (Hubble Ultra DF)