Higgs inflation and gravitational degrees of freedom

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Using what you have

\[ S = \int d^4 x \sqrt{-g} \left( \frac{1 + \xi \phi^2}{2} g^{\alpha \beta} R_{\alpha \beta} - \frac{1}{2} g^{\alpha \beta} \nabla_\alpha \phi \nabla_\beta \phi - V(\phi) \right) \]

\[ V(\phi) = \frac{\lambda}{4} \phi^4 \]

- Inflation with the Standard Model Higgs uses the only known scalar field that may be elementary. (Bezrukov and Shaposhnikov: 0710.3755)

- Non-minimal coupling \( \xi \phi^2 \) makes the Einstein frame potential exponentially flat.
  - The coupling constant \( \xi \) only affects the amplitude.
  - Reheating is known, so no ambiguity in \( N \). (Figueroa et al: 1504.04600)

- The classical predictions are in excellent agreement with observations: \( n_s = 0.96, \ r = 5 \times 10^{-3} \).
When the action is not enough

- Complication: classical low-energy action is not enough to specify the theory.

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- Two sources of ambiguity.
  - Quantum theory: how to calculate loop corrections?
  - General relativity: what are the gravitational degrees of freedom?
Loop-corrected potential

\[ V(\phi) \]

- Different inflationary possibilities:
  - Plateau: apparently not spoiled by loops.
  - Inflection point: can give \( r \sim 0.1 \).
  - False vacuum: new physics needed for graceful exit.
  - Hilltop: under investigation. (Enckell, Enqvist, SR, Tomberg)

\[ \frac{\xi \phi^2}{1 + \xi \phi^2} \]
The many faces of Einstein gravity

\[
S = \int d^4 x \sqrt{-g} \left( \frac{1 + \xi \phi^2}{2} g^{\alpha\beta} R_{\alpha\beta}(g, \partial g, \partial^2 g) - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi - V(\phi) \right)
\]

- Usually the gravitational degrees of freedom are taken to be the metric and its first derivative.

- In the Palatini formalism, the metric and the connection are independent degrees of freedom.

- In the Einstein-Hilbert case, metric and Palatini formalisms are equivalent.

- With a non-minimally coupled scalar field, they give different physical theories. (Bauer and Demir: 0803.2664)
The many faces of Einstein gravity

\[ S = \int d^4x \sqrt{-g} \left( \frac{1 + \xi \phi^2}{2} g^{\alpha\beta} R_{\alpha\beta}(\Gamma, \partial \Gamma) - \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi - V(\phi) \right) \]

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In both cases, the Einstein frame is reached with the conformal transformation $g_{\alpha\beta} = (1 + \xi \phi^2)^{-1} \tilde{g}_{\alpha\beta}$.

In the Palatini case, the conformal transformation does not affect the Ricci tensor.

Therefore we get a different Einstein frame potential.

On the plateau, both give $n_s = 1 \text{-} 2/N = 0.96$, but $r$ is different:

- Metric: $r = 12/N^2 = 5 \times 10^{-3}$
- Palatini: $r = 2/(\xi N^2) = 8 \times 10^{-4}/\xi$ \hspace{1cm} ($\lambda/\xi = 10^{-10}$)
Inflection point inflation: metric vs. Palatini

- Metric formulation range of $r$ is within reach of next generation experiments, Palatini not.
  
  (SR and Wahlman)
  
  (Colour shows the running of the spectral index $\alpha_s = 0.01\pm0.01$.)
Higgs inflation uses only the known particle physics and gravitational degrees of freedom.

Metric formalism value for $r$ will be tested by next generation CMB experiments.

The issue of quantum corrections is not settled.
  - Consistency conditions between cosmology and colliders.

Have to specify the gravitational degrees of freedom.
  - Formulations that are equivalent for Einstein gravity differ when there is a non-minimally coupled scalar: Palatini, teleparallel, ...
  - CMB observations can be used to determine the gravitational degrees of freedom.